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Momentum Vector Control for Spin Recovery

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A rational and robust scheme for obtaining recovery controls for nonlinear flight conditions, such as spins, is presented. In a spin, the loss of control due to the large angular rates and high angle of attack that the airplane experiences could lead to disastrous situations in general aviation. In this study, to arrest rotation and at the same time recover from a spin, the directions and magnitudes of linear and angular momentum vectors are effected to restore their prespin conditions. The problem of computation of the required controls is formulated as a nonlinear programming problem. Simulated time histories are presented and compared to the actual flight-test data for a light, single-engine general aviation aircraft. The proposed method produced effective control time histories for recovery from both a steep and a flat spin, demonstrating the effectiveness of the proposed approach.

Nomenclature

C_{l_β}	=	rolling moment coefficient with respect to sideslip angle
E_S	=	spin kinetic energy
F	=	generic force component
$\boldsymbol{F}$	=	force vector
$\boldsymbol{G}$	=	moment vector
g	=	gravity
$\boldsymbol{H}$	=	angular momentum vector
I	=	moment of inertia
J	=	objective function
K	=	weighting gain
K_L	=	weighting gain for linear momentum vector
K_Ω	=	weighting gain for angular momentum vector
L	=	rolling moment
$\boldsymbol{L}$	=	linear momentum vector
l	=	constraint
M	=	pitching moment
m	=	mass
N	=	yawing moment
P	=	roll rate
Q	=	pitch rate
R	=	yaw rate
t	=	time
t_f	=	final time
t_0	=	initial time
U, V, W	=	velocity vector components in body axes
$\boldsymbol{u}$	=	control vector
$\boldsymbol{V}$	=	linear velocity vector
$\dot{\boldsymbol{V}}$	=	linear acceleration vector
$\boldsymbol{x}$	=	state vector
α	=	angle of attack
β	=	sideslip angle
δ_A	=	aileron deflection
δ_E	=	elevator deflection
δ_R	=	rudder deflection
τ	=	aileron time lag constant
ω	=	magnitude of angular velocity vector
$\boldsymbol{\omega}$	=	angular velocity vector
$\dot{\boldsymbol{\omega}}$	=	angular acceleration vector

Subscripts

A	=	aerodynamic component
B	=	lower bound
C	=	control component
U	=	upper bound
X, x	=	generic component in x axis
Y, y	=	generic component in y axis
Z, z	=	generic component in z axis

Introduction

SINCE the earliest days of aviation, stall/spin has been one of the leading causes of accidents and fatalities in general aviation. A spin, or "an aggravated stall that results in autorotation,"[1] is, in reality, one of the most complicated motions an airplane can get into due to its highly coupled motion in all axes and separated flow around the spinning airplane. The most severe problem associated with stall/spin is the loss of control caused by the combination of separated flow on stalled wings and control surfaces and high rotational rates. This loss of control in airplane spins causes an inexperienced pilot to be disoriented, sometimes leading to catastrophic results.

During the 1970s and 1980s, extensive research was conducted to analyze and characterize the nature of spins, most of which led to the determination of alternative airplane configuration or mass distribution intended to render an aircraft spin resistant, or easy to recover from a spin. The concept of a notched leading edge emerged and was proven to be successful in making the aircraft difficult to spin by delaying tip stall.[2] These research efforts involved extensive flight testing,[3] as well as static wind-tunnel and spin-tunnel testing on a light, single-engine general aviation airplane.[4–6] For instance, Ref. 3 reports a large amount of flight-testing to analyze the characteristics of spin entry and recovery with different combinations of airplane configurations, such as tail configuration, mass distribution, position of center of gravity, and different combinations of control inputs to enter the spin and to recover. For the purpose of explanation of the nature of a developed spin, the equilibrium spin analysis theory was developed.[7–12] Equilibrium spin analysis was the first theoretical approach applied to spins that are fully developed and was successful in analyzing spins. These works mainly focused on the determination of the state at which equilibrium exists. The three moment equations, based on the equilibrium of aerodynamic

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moments and inertial moments, played a dominant role in the analysis of the equilibrium of the developed spins. There are many factors affecting spin characteristics of a particular airplane. Much of the research effort concluded that the characteristics of spins are mainly affected by mass distribution and tail configuration of an airplane.

There were also several investigations related to the recovery phase.[13–15] Some of the research was directed toward analysis of the effect of geometric configuration on the early stage of recovery, such as the effects of horizontal tail configurations and inertia moments. Initial estimation of required moments and/or effects of various configurations of airplane for spin recovery were also presented.

However, the high angular rates of a spinning airplane make the pilot disoriented and, thus, limit the pilot's ability to apply the proper spin recovery procedures. This is especially true for inexperienced general aviation pilots. This necessitates the use of suitable automatic spin recovery schemes. In Ref. 16, functional optimization was attempted to determine optimal controls for effective recovery from a flat spin by minimization of the time derivatives of angle of attack, sideslip angle, and angular rates. The angular momentum suppression method[17] uses damping moments to suppress the angular momentum vector when an F/A-18D aircraft is performing the falling leaf motion. The falling leaf is a motion that, like spins, shows strong all-axes coupling such as in-phase roll and yaw rates with rapid angle of attack and sideslip traverses. The study proposed a concept that the direct suppression of the angular momentum vector is an effective way to arrest rotation when an airplane experiences high angle of attack and angular rates. The successful application of this concept to spin recovery is presented in Ref. 18. In this work, an inverse solution approach was proposed to arrest the rotation of a fully developed spin. This approach was shown to be a valid method for recovering from a spin.

In the studies in Refs. 16–18, the efforts were directed at slowing angular rates to bring the airplane into controllable conditions and reduce the angle of attack to a region where in aerodynamic controls are effective. The emphasis of the present study is placed on the full recovery phase. In this study, the control of both linear and angular momentum vectors is proposed as an automated spin recovery scheme to arrest rotation and also to recover the linear velocity vector (or equivalently, the dynamic pressure, angle of attack, and sideslip angle) of a spinning airplane. The main idea of the momentum vector control is to reduce the kinetic energy of the airplane in a spin, which differs from its prespin condition due to the rotation and associated changes of velocities in each body axis. The kinetic energy due to the deviation from steady-state and/or prespin conditions is expressed in terms of linear and angular momentum vectors. In the arrest of rotation and effort to bring the airplane to its prespin conditions, the directions and magnitudes of those vectors are effected by aerodynamic controls. This is formulated as a nonlinear programming problem. The sequential quadratic programming method was chosen due to its computational efficiency to produce the necessary controls for recovery.

Momentum Vector Control

Formulation of Automatic Spin Recovery as a Nonlinear Optimal Control Problem

Spin recovery schemes, conventional spin recovery procedures, or deployment of a spin-chute, are intended to arrest the rotation and/or bring the airplane into the region where the aerodynamic controls are effective. For instance, the large drag of a deployed spinchute generates moment that will rotate the nose down, hence, reducing the angle of attack of a spinning airplane. In general, antispin schemes reduce the kinetic energy of a spinning airplane directly or indirectly. In this study, the kinetic energy of a spinning airplane is broken down into two parts: 1) the kinetic energy due to the spinning airplane and 2) the effectiveness of anti-spin schemes applied for recovering from a spin. This is formulated as

$$E_S(t_f) = E_S(t_0) + \underbrace{\int_{t_0}^{t_f} \frac{\mathrm{d}E_S(t)}{\mathrm{d}t}\,\mathrm{d}t}_{\text{effectiveness of antispin scheme}} \tag{1}$$

where t_f is arbitrary. For the spin recovery scheme to be successful, the spin kinetic energy at t_f needs to be close to zero as possible, that is, $E_S(t_f) \cong 0$.

Equation (1) is rewritten as a discrete form for a fixed time interval Δt and $t_0 = 0$, such that,

$$E_S(N) = E_S(0) + \sum_{i=0}^{N-1} \frac{\mathrm{d}E_S(i)}{\mathrm{d}t}\Delta t \tag{2}$$

where t_f is approximated as $t_f = N \cdot \Delta t$. The equation can be solved to find the final value of $E_S(N)$ in terms of a difference equation

$$E_S(i+1) = E_S(i) + \frac{\mathrm{d}E_S(i)}{\mathrm{d}t}\Delta t, \qquad i = 0, \ldots, N-1 \tag{3}$$

For the spin kinetic energy $E_S(N)$ at the final time to be zero, the intermediate magnitudes of $E_S(i+1)$ need to be reduced. It may not be physically possible to reduce the magnitude of $E_S(i+1)$ to zero at a single time step, but assume that any antispin scheme applied will attempt to reduce the magnitude of $E_S(i+1)$ close to zero. The objective here is to attempt to reduce the magnitude of $E_S(i+1)$ as close to zero as possible at every time step, finding the proper rate of change of the spin kinetic energy. For the magnitude of $E_S(i+1)$ to be zero, the following relationship needs to be satisfied at every time interval:

$$E_S(i) + \frac{\mathrm{d}E_S(i)}{\mathrm{d}t}\Delta t = 0, \qquad i = 0, \ldots, N-1 \tag{4}$$

The spin kinetic energy of a spinning airplane is divided into two parts, the part due to the angular rates of the spinning airplane and the part due to the perturbation in linear velocities in each body axis from a prespin condition. In level flight, the kinetic energy due to the rotation of an airplane is negligible. However, when the airplane starts rotating, in addition to the kinetic energy due to rotation, the kinetic energy due to the change of its angles of attack and sideslip is no longer trivial. The linear velocity and the linear momentum vectors perturbed from a level flight condition are denoted as $\boldsymbol{V}$ and $\boldsymbol{L}$. If the kinetic energy of a spinning airplane is expressed as a function of the linear and angular velocity and momentum vectors, the spin kinetic energy and its time derivatives are expressed as

$$E_S = \frac{1}{2}\boldsymbol{\omega}\cdot\boldsymbol{H} + \frac{1}{2}\boldsymbol{V}\cdot\boldsymbol{L} \tag{5}$$

$$\frac{\mathrm{d}E_S}{\mathrm{d}t} = \frac{1}{2}(\dot{\boldsymbol{\omega}}\cdot\boldsymbol{H} + \boldsymbol{\omega}\cdot\boldsymbol{G} + \dot{\boldsymbol{V}}\cdot\boldsymbol{L} + \boldsymbol{V}\cdot\boldsymbol{F}) \tag{6}$$

When an antispin scheme is applied for recovering from a spin, Eq. (4) becomes

$$\boldsymbol{\omega}\cdot\boldsymbol{H} + (\dot{\boldsymbol{\omega}}\cdot\boldsymbol{H} + \boldsymbol{\omega}\cdot\boldsymbol{G})\Delta t + \boldsymbol{V}\cdot\boldsymbol{L} + (\dot{\boldsymbol{V}}\cdot\boldsymbol{L} + \boldsymbol{V}\cdot\boldsymbol{F})\Delta t = 0 \tag{7}$$

When $K = 1/\Delta t$ is defined and rearranged, Eq. (7) becomes

$$\underbrace{[\dot{\boldsymbol{\omega}}\cdot\boldsymbol{H} + \boldsymbol{\omega}\cdot(K\boldsymbol{H}+\boldsymbol{G})]}_{\text{rotational}} + \underbrace{[\dot{\boldsymbol{V}}\cdot\boldsymbol{L} + \boldsymbol{V}\cdot(K\boldsymbol{L}+\boldsymbol{F})]}_{\text{translational}} = 0 \tag{8}$$

The spin kinetic energy in Eq. (8) is separated into two parts corresponding to rotational and translational modes, and each part is allowed to go to zero at each time interval such that

$$\dot{\boldsymbol{\omega}}\cdot\boldsymbol{H} + \boldsymbol{\omega}\cdot(K\boldsymbol{H}+\boldsymbol{G}) = 0 \tag{9}$$

$$\dot{\boldsymbol{V}}\cdot\boldsymbol{L} + \boldsymbol{V}\cdot(K\boldsymbol{L}+\boldsymbol{F}) = 0 \tag{10}$$

If $\boldsymbol{H} = K\boldsymbol{H} + \boldsymbol{G}$ and $\boldsymbol{L} = K\boldsymbol{L} + \boldsymbol{F}$, the differential equations for the rate of change of the magnitudes of the linear and angular velocity vectors can guarantee that the eigenvalues of the differential equations are negative within the range of physically meaningful angles, in this case, between 0 and 90 deg. For instance, if the angular velocity vector $\boldsymbol{\omega}$ is orthogonal to the vector $K\boldsymbol{H} + \boldsymbol{G}$, there exists a constant magnitude of $\boldsymbol{\omega}$ (equivalent to a zero eigenvalue), in which case any external moments applied to a spinning airplane are able to rotate a set of two vectors to a new orientation but cannot reduce the magnitude of the angular velocity vector. From this, we can state the following: If the angular momentum vector $\boldsymbol{H}$ is equal to the vector $K\boldsymbol{H}+\boldsymbol{G}$, that is, $(K-1)\boldsymbol{H}+\boldsymbol{G}=0$, and the linear momentum vector $\boldsymbol{L}$ is equal to the vector $K\boldsymbol{L}+\boldsymbol{F}$, that is, $(K-1)\boldsymbol{L}+\boldsymbol{F}=0$, the magnitudes of linear and angular velocity vectors become zero if there exist proper external force and moment vectors $\boldsymbol{F}$ and $\boldsymbol{G}$.

Optimal Control Approach for Momentum Vector Control

The preceding section presents primary guidelines for the design of an automated spin (or other highly nonlinear flight condition) recovery scheme that operates in a regulator mode. The scheme also needs to reduce the following vector to zero:

$$\boldsymbol{J}=\begin{bmatrix}(K-1)\boldsymbol{L}+\boldsymbol{F}\\(K-1)\boldsymbol{H}+\boldsymbol{G}\end{bmatrix} \tag{11}$$

This is to find the control forces and moments required to reduce the magnitudes of the linear and angular momentum vectors, weighted by the gain $(K-1)$, respectively. This scheme is referred to as the momentum vector control. The goal of the momentum vector control is not only the reduction of the magnitudes of the momentum vectors applied to a spinning airplane, but also changing the direction of those vectors where the force and moment vectors are most effective in reducing their magnitudes. This means that the force and moment components should be applied parallel and opposite to the linear and angular momentum vector components to reduce the magnitude of the given vector in Eq. (11).

To achieve this objective, Eq. (11) is modified as a suitable form to a nonlinear programming problem with the assumption that the body axes coincide with the principal axes of the airplane. Also, because the force and moment vectors applied to an airplane are composed of the state-dependent part, which depends strictly on states, and the control-dependent part, which depends on states as well as controls, the scalar objective functions is written as

$$J=\boldsymbol{J}^T\cdot\boldsymbol{J}=\left(K_LmU+F_{X_A}+F_{X_C}\right)^2+\left(K_LmV+F_{Y_A}+F_{Y_C}\right)^2$$
$$+\left(K_LmW+F_{Z_A}+F_{Z_C}\right)^2+(K_\Omega I_XP+L_A+L_C)^2$$
$$+(K_\Omega I_YQ+M_A+M_C)^2(K_\Omega I_ZR+N_A+N_C)^2 \tag{12}$$

where $(K-1)$ in Eq. (11) has been replaced by K_L and K_Ω, respectively. Now, the goal of the automated spin recovery scheme is to find a control vector $\boldsymbol{u}=\{\delta_A,\delta_E,\delta_R\}$, which minimizes the given objective function:

$$\text{Find}\,\boldsymbol{u}\quad\text{for}\quad\min J,\qquad l_B\le\boldsymbol{u}\le l_U \tag{13}$$

The formulation is a constrained nonlinear programming problem subject to the upper and lower bounds for controls. Magnitudes of the controls that are within these bounds occur frequently and are necessary to obtain the desired control action.

Initial simulation results indicated that a variable weighting gain for the rotational part will result in a smoother recovery. Thus,

$$K_\Omega=K_0\cdot\omega \tag{14}$$

where $\omega=\sqrt{(P^2+Q^2+R^2)}$. In this manner, the variable gain K_Ω is applied according to the degree of rotation. However, for the gain K_L, a fixed value

$$K_L=K_0 \tag{15}$$

was found to be appropriate. This leaves the constant weighting gain K_0, which is used for both K_L and K_Ω, as an arbitrary parameter to be evaluated in conjunction with system performance. K_0 is similar to a feedback gain in linear control systems and can be chosen for the best performance of the given systems. The gain value was selected by trying different simulation runs in a manner similar to that explained in Ref. 18. It mainly affects the recovery time and some controls' oscillations. The way to obtain a proper value of K_0 is analogous to the practice used in conventional controller designs, that is, by trial and error, through inspection of the time history results. In this study, the value of gain K_0 is chosen such that the simulated time histories closely match the flight data. If the angular momentum part is considered alone, the objective function in Eq. (12) reduces to the form used in Ref. 18.

Momentum Control Augmentation System

Figure 1 shows the momentum vector control concept in a regulator mode to illustrate how to obtain control deflections to control the linear velocities and angular rates. The objective is to produce a control vector $\boldsymbol{u}$ that will reduce the magnitude of the linear and angular momentum vectors perturbed from level flight conditions, $\boldsymbol{H}=\boldsymbol{I}\cdot\boldsymbol{\omega}$ and $\boldsymbol{L}=m\boldsymbol{V}$. The angular velocity $\boldsymbol{\omega}$ and the linear velocity $\boldsymbol{V}$ are obtained or estimated from measurements. To reduce $\boldsymbol{H}$ and $\boldsymbol{L}$, a set of aerodynamic forces and moments $\boldsymbol{G}=K_\Omega\boldsymbol{H}$ and $\boldsymbol{F}=K_L$, with the gains $K_\Omega<0$ and $K_L<0$, are desired. The part of each of $\boldsymbol{G}$ and $\boldsymbol{F}$ and $\boldsymbol{G}_S$ and $\boldsymbol{F}_S$ depend strictly on the state $\{\boldsymbol{V},\boldsymbol{\omega}\}$. They are referred to as the state-dependent aerodynamic forces and moment. The other parts

$$\boldsymbol{F}_C=\boldsymbol{F}-\boldsymbol{F}_S=K_L\boldsymbol{L}-\boldsymbol{F}_S \tag{16}$$

$$\boldsymbol{G}_C=\boldsymbol{G}-\boldsymbol{G}_S=K_\Omega\boldsymbol{H}-\boldsymbol{G}_S \tag{17}$$

depend additionally on the controls. They are referred to as the control-dependent aerodynamic forces and moments.

In the momentum vector control augmentation system, to find the inverse solution for $\boldsymbol{u}$, the control surface deflections must achieve the necessary control forces and moments. This is performed by means of an optimization algorithm. The optimization algorithm

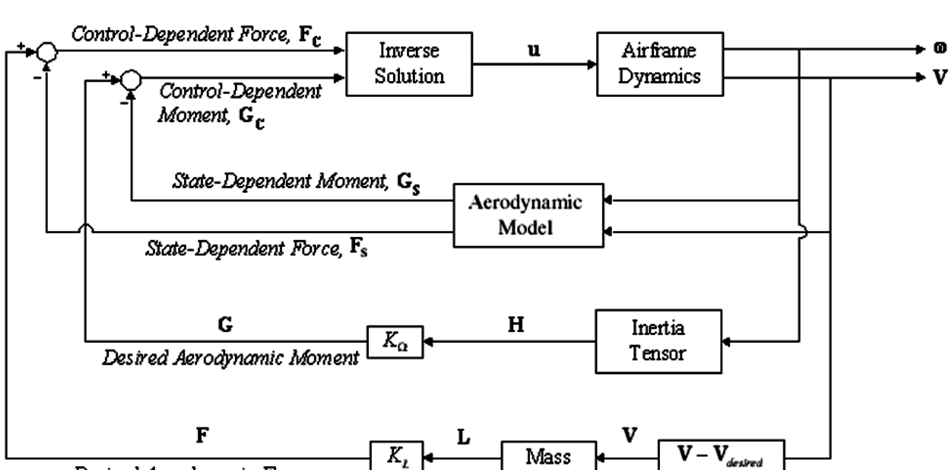


Fig. 1 Momentum vector control augmentation system.

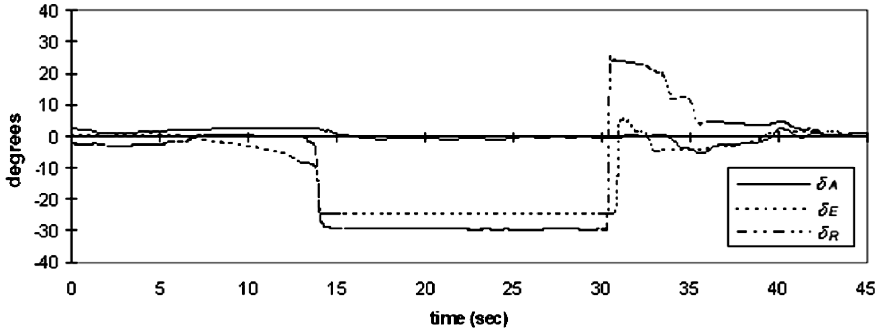


Fig. 2 Measured control time histories for spin *G* simulation result.

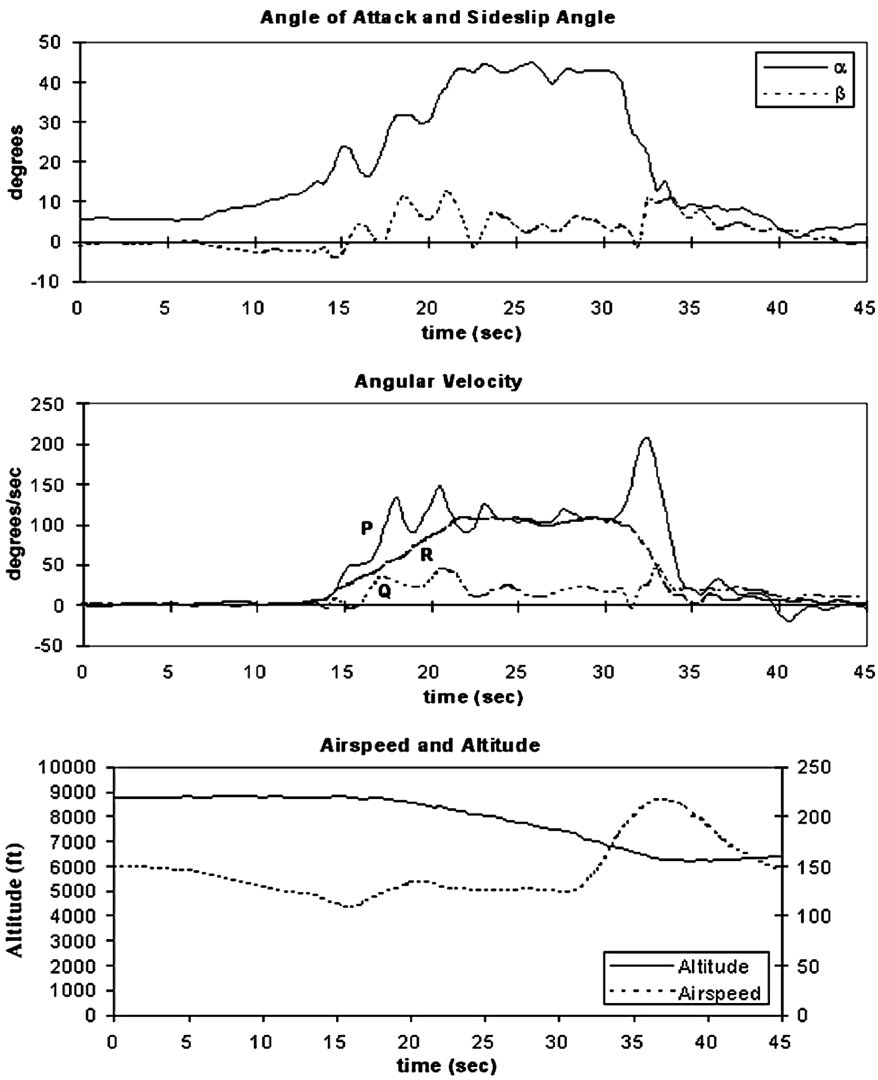


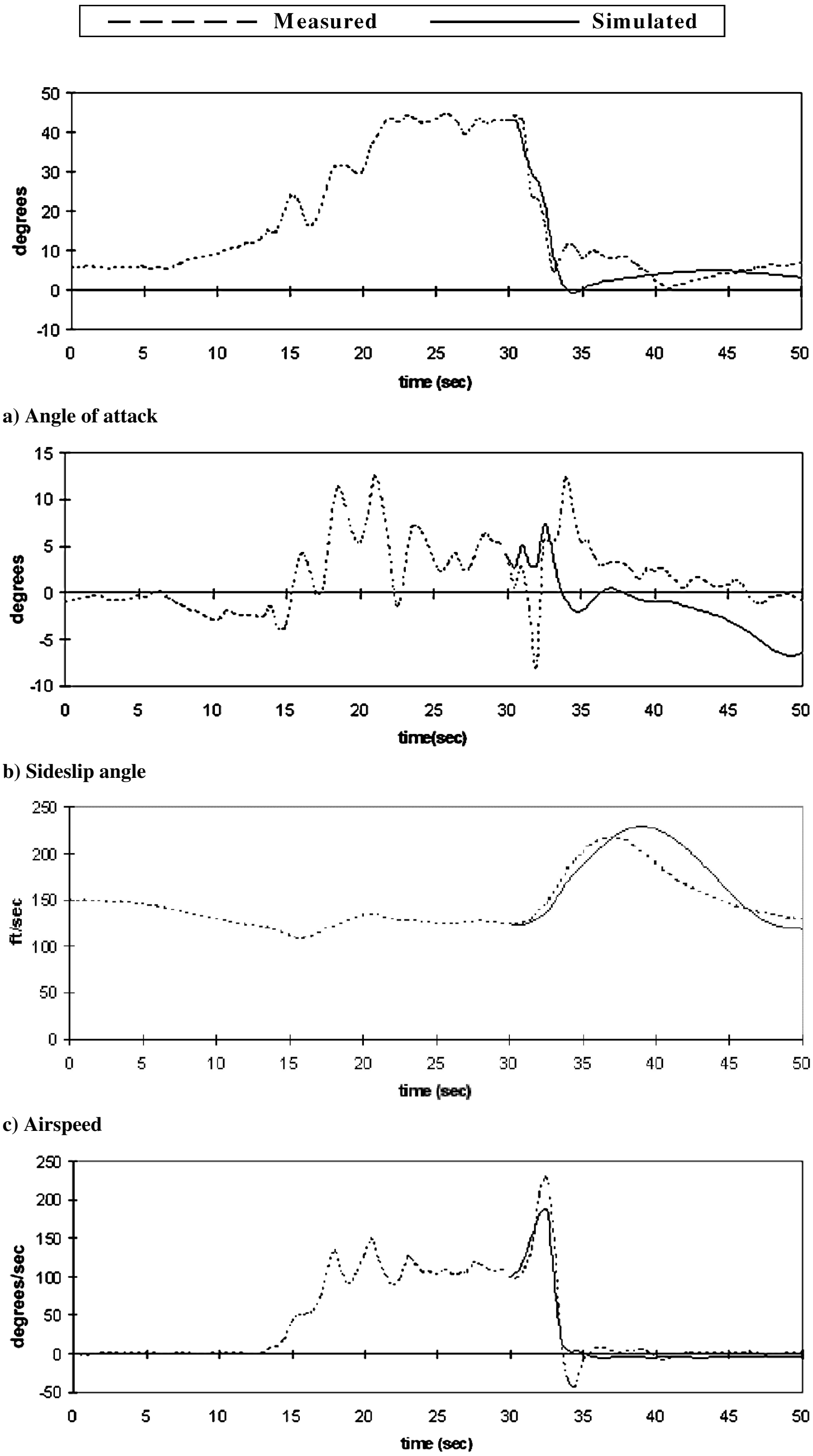
Fig. 3 Spin recovery simulations with measured controls for spin *G*.

finds the control forces and moments that make the desired aerodynamic forces and moments, $K_L \boldsymbol{L}$ and $K_\Omega \boldsymbol{H}$, approach zero.

Aileron Servo Model

A standard pilot's technique for recovery uses a neutralized aileron with full antispin rudder and full trailing-edge-down elevator. Flight tests for spin recovery in Ref. 3 show that this procedure was successful in the arrest of rotation and recovery of attitude for moderate spins. However, measured control deflections time histories in Fig. 2, which are the control deflections that the pilot produced to recover from the flight-test spin maneuver (designated as spin G), imply that there the aileron inputs are needed during the spin recovery. In the flight test, the pilot needed to apply the aileron inputs to offset undesirable gyroscopic and aerodynamic effects caused by the coupling between pitch and yaw. Because of the minimal aerodynamic effect of the aileron during the spin, this aileron deflection application is delayed until the angular rates are reduced considerably. In the optimal control algorithm, the effect of aileron during the spin recovery was modeled to attenuate unnecessary rapid motion in roll axis caused by the less effective rolling moment of the aileron. This was done by the introduction of an aileron time lag:

$$\dot{\delta}_A = -(1/\tau)\delta_A + \delta_{A_c} \tag{18}$$



a) Angle of attack

b) Sideslip angle

c) Airspeed

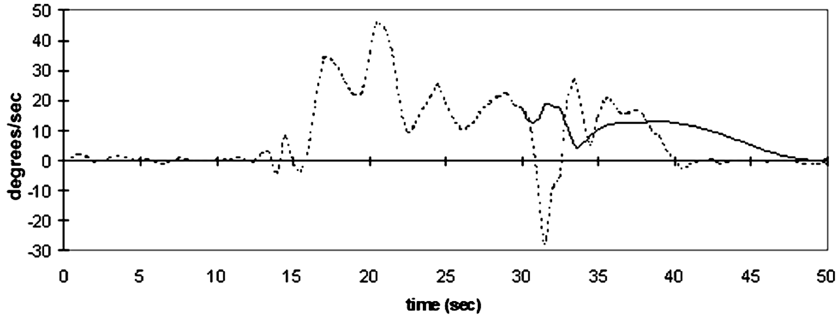
d) Roll rate

Fig. 4 Spin recovery for spin *G* with spin *G* regression.

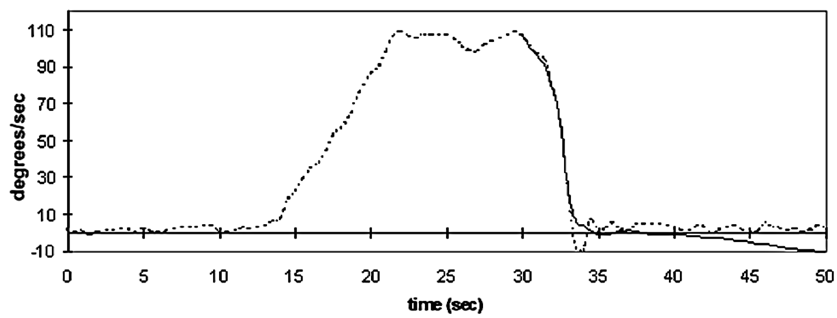
where δ_A is the current aileron deflection, and δ_{A_c} is the aileron deflection command calculated by the automatic spin recovery algorithm. The aileron servo differential equation induces a lag in the current aileron deflection leading to a smoothing of aileron action. This choice was based on the typical lag used in general aviation airplanes, which is from about 0.1 to 0.3 s (in this study, 0.2 s), and is a result of acceptable control time histories. This aileron servo model can be more elaborate and introduce the response lag of the states to aileron inputs. Without the attenuation, the aileron motion that results from the computed controls displayed a oscillatory (noisy) character and was deemed unacceptable. This servo model actually acts as a filter, which will be present if the system is implemented in an aircraft. The elevator and rudder will also be actuated through a lag, but it was not necessary to include it here because their inputs were not noisy.

Algorithm Description

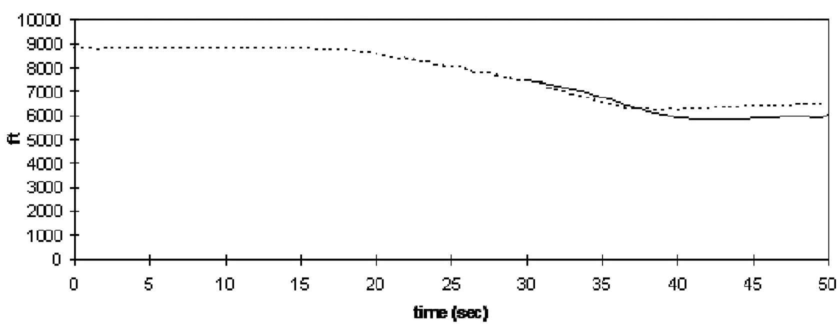
The algorithm consists of two parts, namely, the estimation of state- and control-dependent aerodynamic forces and the moments, and the computation of recovery controls. At each time step, the



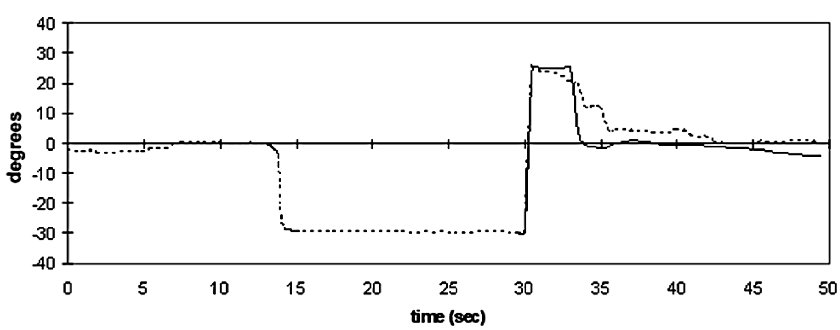
e) Pitch rate



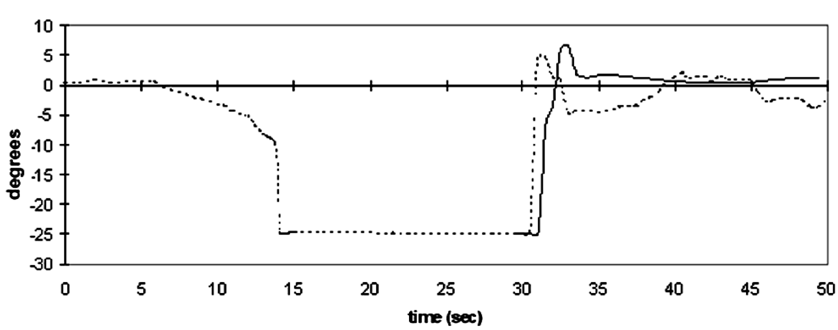
f) Yaw rate



g) Altitude

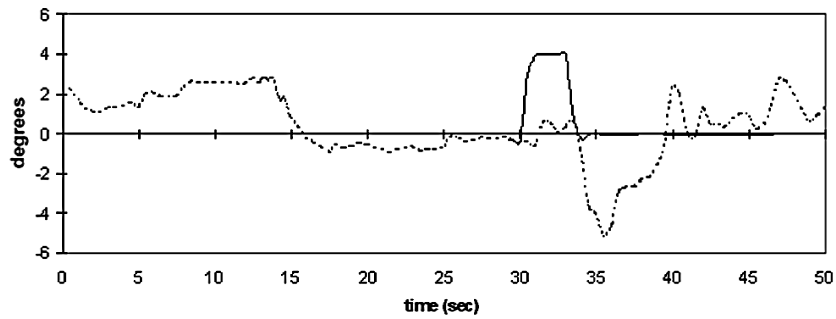


h) Rudder deflection



i) Elevator deflection

Fig. 4 Spin recovery for spin *G* with spin *G* regression (continued).



j) Aileron deflection

Fig. 4 Spin recovery for spin *G* with spin *G* regression (continued).

state-dependent forces and moments are calculated by the use of the kinematic relationship and the aerodynamic coefficients modeled in Refs. 19–21. The control derivatives are estimated base on current angle of attack and the models based on measured static wind-tunnel data.[4]

The accuracy of control forces and moments used in this work is limited because the wind-tunnel testing used to obtain their model was conducted for more general purposes. For example, the effect of downwash and sidewash at the tail could not account for the rotation. Also, the secondary effects, such as interference and cross coupling, were not accounted because the data did not include such effects. The aerodynamic models are valid for specific spins because they are based on the regression models preformed on each set of flight data. If one regression model is applied to a different spin, it introduces more errors, for instance, the application of spin G regression to spin P. However, the aerodynamic and control models used in this study make it possible to analyze the aerodynamic and dynamic characteristics of highly nonlinear aircraft motion with reasonable accuracy. To validate the aerodynamic and control models used in this study, the simulated time histories based on the flight-test control inputs (designated as spin G) are presented in Fig. 3. In Fig. 3 the state time histories are repeated from flight tests up to $t = 30$ s. Thereafter, the flight-test control inputs are used in the simulation to obtain the state past 30 s. This is intended to show the validity of the aerodynamic model used.

Required optimal controls are calculated when the objective function given in Eq. (12) in minimized. To find the optimal controls, the constrained nonlinear programming problem is solved with NPSOL.[22] To determine the time response, the nonlinear six-degrees-of-freedom equations of motion are solved by the use of the computed control surfaces.

Simulated Automatic Spin Recovery

In this study, two spins in Ref. 3, designated as spin G and spin P, are considered to demonstrate the validity of the proposed approach. These two spins are considered typical spins that a general aviation airplane might enter. Two regression results for spins G and P given in Ref. 21 are used to calculate the state-dependent aerodynamic forces and moments.

To compare the computed controls and trajectories with the flight-test data, the algorithm starts at the same time as the pilot initiated the spin recovery. In other words, the initial values of states and controls are those of the states and the controls in the flight-test data at this time. In this algorithm, the recovery controls are computed repeatedly at 0.1-s intervals. The weighting gain K_0 used in the objective function is chosen as 3.0 for both cases for good performance of the system. The limits on the control surface deflections are the same physical limits of controls for the actual flight test.

Spin Recovery for Spin *G* Using Spin *G* Regression Result

Calculated control surface deflections required for recovery from the spin and simulated time histories are presented in Fig. 4. The spin G regression result is used to estimate the state-dependent aerodynamic forces and moments. The computed optimal controls are shown to be successful in arresting rotation, restoring angles of attack and sideslip to their steady-state conditions, and arresting the loss of altitude. The total drop in altitude is similar to the measured data. Also, the simulated time histories show similar trends and magnitudes to measured ones. The aileron is deflected to reduce the rolling moment caused by the coupling between pitch and yaw during the recovery.

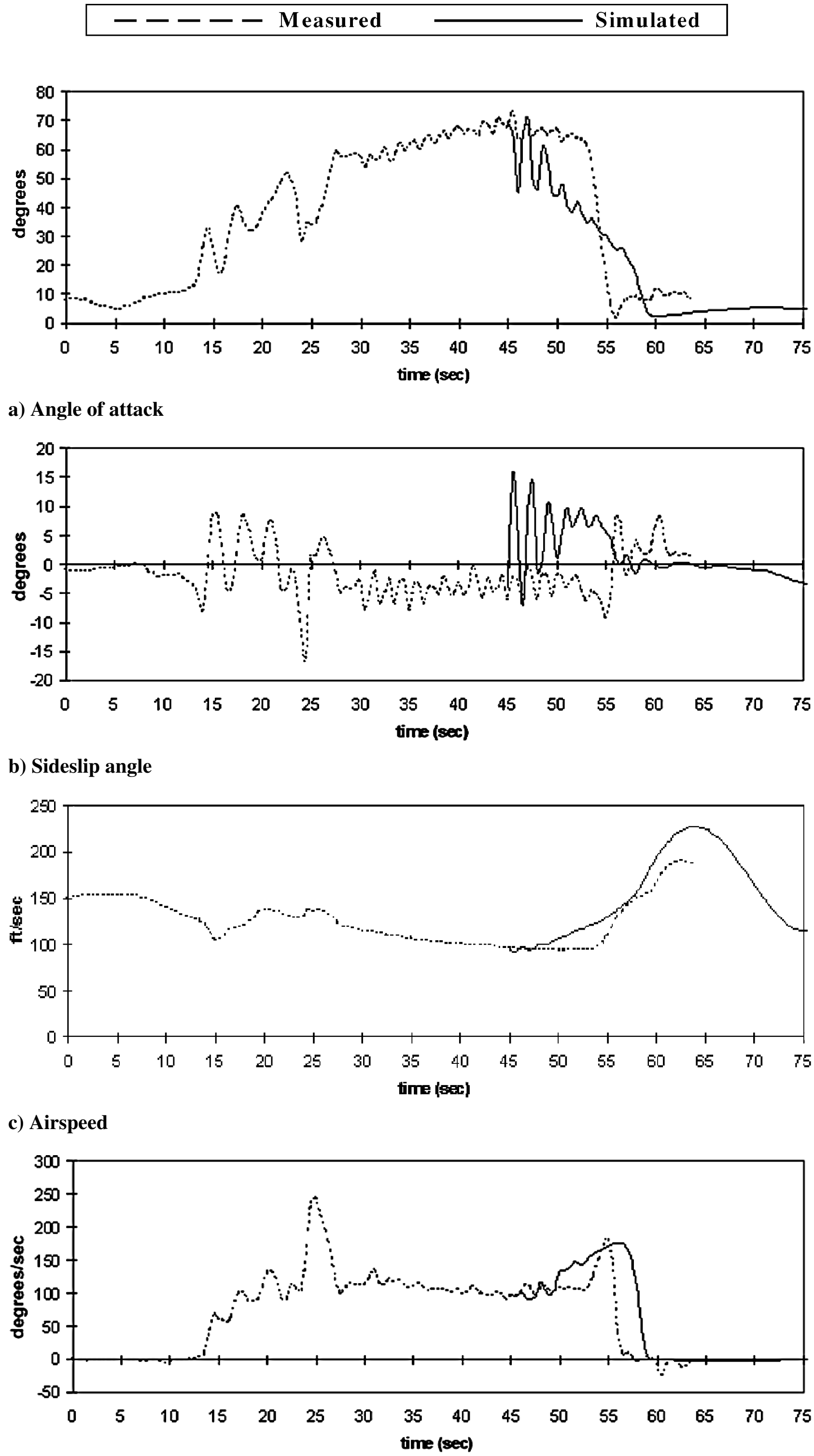
The relationship between the angular momentum vector and the total moment vector applied reveals that when an airplane is in a spin, direct application of external moments does not suppress the rotation of the airplane immediately because the total moment vector is not parallel to the angular momentum vector. To arrest rotation, the total moment vector should be initially parallel and opposite to the angular momentum vector. This was true for both cases of the flight test and the simulated spin recovery. After the angular momentum vector was parallel to the total moment vector, the total moment vector was able to suppress the angular momentum vector. The control moments played a role to make the total moment vector parallel to the angular momentum vector of a spinning airplane because, without the control moments, the total moment vector cannot be always parallel to the angular momentum vector.

The Euler angles can be easily recovered when the rotation is stopped and the airspeed and angle of attack are restored. For instance, in spin G, the roll and pitch angles were approximately -40 and 20 deg when the spin recovery was completed, which can easily be corrected by pilots, or by the introduction of conventional attitude control. Because the proposed approach does not include a scheme to restore the angles, but only the rates, the roll angle will not go back to zero quickly on its own unless C_{l_β} is very negative.

Spin Recovery for Spin *P* Using Spin *P* Regression Result

A simulation of spin P with spin P regression is presented in Fig. 5. Important states, such as angle of attack, sideslip angle, and angular rates, are stabilized 25 s after the algorithm starts and the recovery is completed within 30 s. In flight testing, to enter a flat spin, that is, in which the angular rates are stabilized, the test pilot applied a certain sequence of elevator deflections. A flat spin is difficult to recover from due to high angle of attack, and, as a result, standard recovery controls are not usually effective. The calculated elevator deflections required for recovery from the spin show the reversed order of the sequence of elevator deflections that the pilot applied to enter the flat spin. The application of the sequence of trailing-edge-down and-up elevator deflections that hold the full antispin rudder make the pitch rate unstable and, therefore, easier to recover. The spin recovery is initiated around 50 s after the pitch rate becomes unstable, which makes the stable spin unstable. After the stable spin becomes unstable, the simulated control time histories display in a manner similar to the spin G recovery that uses spin G regression. However, in flight testing, the test pilot applied the nominal spin recovery procedure without attempting to make the stable spin unstable, and, as a result, the recovery procedure was not successful.

In spin P, the total moment vector could not orient the angular momentum vector parallel to itself until the spinchute was deployed.



a) Angle of attack

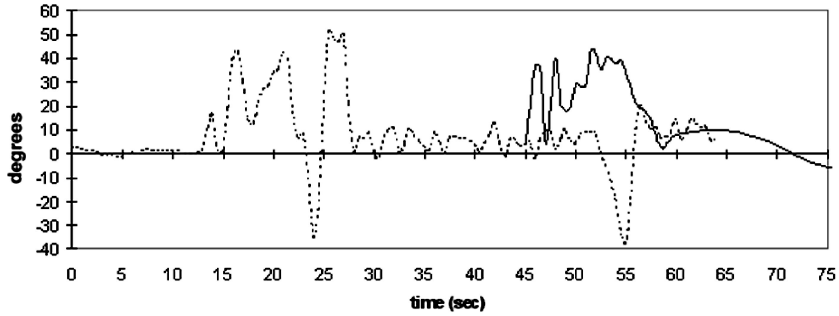
b) Sideslip angle

c) Airspeed

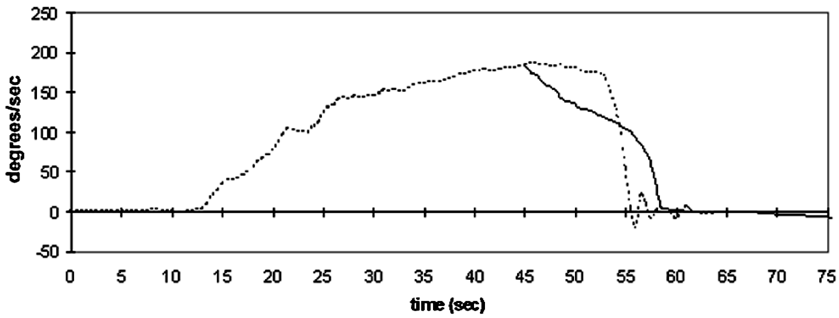
d) Roll rate

Fig. 5 Spin recovery for spin *P* with spin *P* regression.

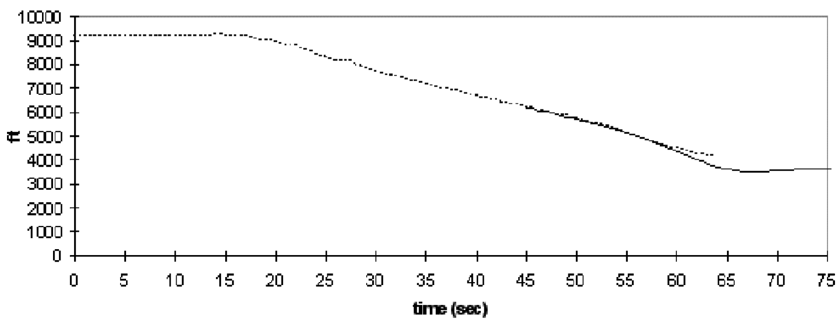
The angular momentum vector showed a stable precession similar to the motion of a top. After the gyration of the angular momentum vector became unstable, the control moments effectively reduced the magnitude of the angular momentum vector that had the total moment vector parallel to the angular momentum vector. When the recovery was completed, the attitude of the airplane reached 30 and 20 deg in roll and pitch attitudes, in the same manner as the spin G recovery. The algorithm was shown to be successful in its recovery from the stable spin, from which the test pilot seemed unable to recover. In spin G, when the control surfaces are simply neutralized, the aircraft slowly recovers on its own. However, in spin P, this is not the case: Neutralization of the controls does not reduce the momentum deviations. These maintain large, steady values, and only the application of proper combined and coordinated control time histories was able to affect the recovery. The algorithm also found the sequence of the elevator deflections necessary to make the stable spin unstable. This was accomplished by deployment of the spinchute in the flight test.



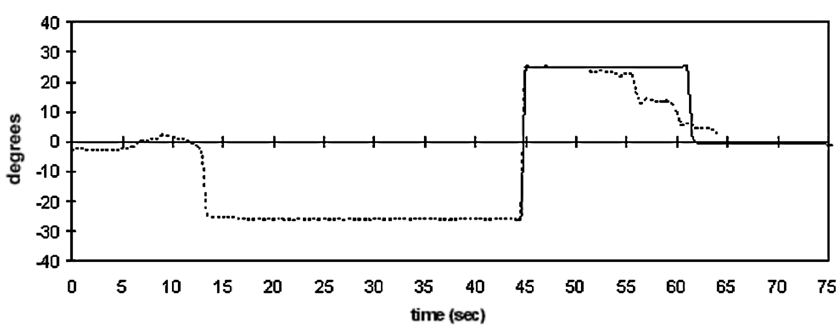
e) Pitch rate



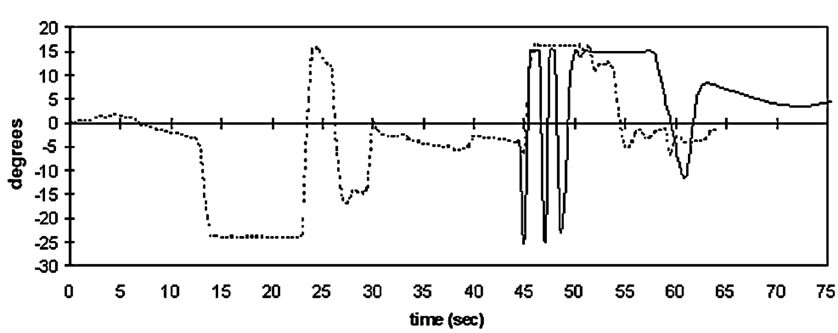
f) Yaw rate



g) Altitude

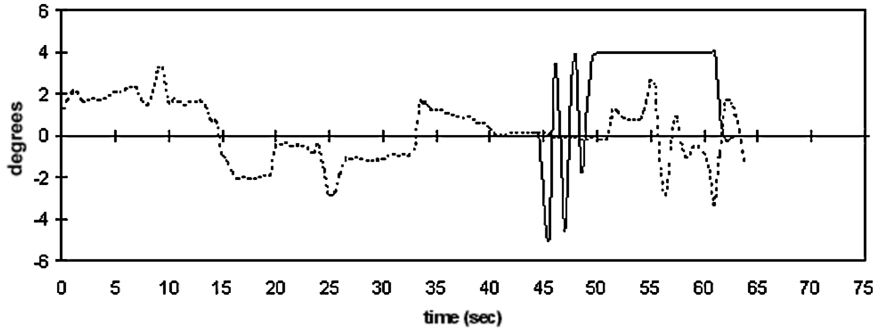


h) Rudder deflection



i) Elevator deflection

Fig. 5 Spin recovery for spin *P* with spin *P* regression (continued).



j) Aileron deflection

Fig. 5 Spin recovery for spin *P* with spin *P* regression (continued).

Summary

In this study, a scheme for momentum vector control under highly complex and nonlinear flight conditions is presented for the purpose of enabling recovery from maneuvers, entered intentionally or inadvertently, in which large angles of attack and/or sideslip and large angular rates exist. The proposed momentum (both linear and angular) vector control approach for automated spin recovery was successful in the arrest of the high rotation rates and in the recovery of the attitude of the spinning airplane, where the linear velocity vector (or, equivalently, the dynamic pressure, angle of attack, and sideslip angle) is returned to prestall-spin. The simulated results and comparison with the flight-test data demonstrate the validity of this approach. The result of this rational and robust scheme was shown to be successful in simulations in which the full nonlinear aerodynamic model is taken into account. The implementation of such a method can be performed by either the provision of an advisory to the pilot as to what control input sequence is required, or the method may be implemented as an automatic controller that will produce these controls.

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